Uitwerking tentamen Golven en Optica, 20/11/97

1.a) 

$F_1 = F_2 \sin\theta \approx Mg\theta$ $\quad v = \frac{d}{dt} L\theta = L\frac{d\theta}{dt} \quad \rightarrow \quad a = L\frac{d^2\theta}{dt^2}$

$\Rightarrow ML\frac{d^2\theta}{dt^2} + M\gamma L\frac{d\theta}{dt} + Mg\theta = 0 \Rightarrow \frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \frac{g}{L}\theta = 0$

$\Rightarrow$ met $\gamma = 0$ en $\frac{d^2\theta}{dt^2} + \omega_0^2\theta = 0 \Rightarrow \omega_0 = \sqrt{\frac{g}{L}}$

Energie: $E_k = \frac{1}{2}Mv^2 = \frac{1}{2}ML^2\left(\frac{d\theta}{dt}\right)^2 \quad E_p = Mg(L - L\cos\theta) \approx MgL\frac{1}{2}\theta^2$

$\Rightarrow \frac{1}{2}ML^2\left(\frac{d\theta}{dt}\right)^2 + \frac{1}{2}MgL\theta^2 = E(1 - e^{-\gamma t})$; $\gamma = 0 \rightarrow \omega_0^2$: verhouding voorfactoren $\rightarrow$

$\Rightarrow \omega_0^2 = \frac{\frac{1}{2}MgL}{\frac{1}{2}ML^2} = \frac{g}{L} \Rightarrow \omega_0 = \sqrt{\frac{g}{L}}$

b) $\frac{d^2\theta}{dt^2} + \gamma\frac{d\theta}{dt} + \omega_0^2\theta = 0$ zie collegesheet: probeer $\theta = Ce^{qt} \Rightarrow q^2 + \gamma q + \omega_0^2 = 0$

$\Rightarrow q = -\frac{1}{2}\gamma \pm \sqrt{\frac{1}{4}\gamma^2 - \omega_0^2}$; ook mogelijk $\theta = Cte^{qt} \Rightarrow q = \frac{\gamma}{2} \quad \omega_0^2 = \gamma^2/4$

1) Licht gedempt $\omega_0^2 > \frac{1}{4}\gamma^2 \Rightarrow q = -\frac{1}{2}\gamma \pm i\omega \Rightarrow \theta = C_1 e^{-\gamma/2\, t}e^{i\omega t} + C_2 e^{-\gamma/2\, t}e^{-i\omega t}$

met θ reëel $\Rightarrow C_2 = C_1^* \rightarrow \theta = Ae^{-\gamma/2\, t}\cos(\omega t + \alpha)$

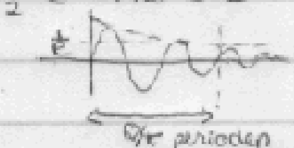
2) Overgedempt: zie collegesheet: $\theta = C_1 e^{(-\frac{1}{2}\gamma + \beta)t} + C_2 e^{(-\frac{1}{2}\gamma - \beta)t}$ met $\beta = \sqrt{\frac{1}{4}\gamma^2 - \omega_0^2}$

3) Kritisch gedempt: $q = -\frac{\gamma}{2} \rightarrow \theta = (A + Bt)e^{-\frac{\gamma}{2}t}$

c) $\theta = Ae^{-\frac{\gamma}{2}t}\cos(\omega t + \alpha) \quad \frac{d}{dt}\theta = -A\frac{\gamma}{2}e^{-\frac{\gamma}{2}t}\cos(\omega t + \alpha) - Ae^{-\frac{\gamma}{2}t}\omega\sin(\omega t + \alpha)$

$\theta(t=0) = A\cos\alpha = 0 \Rightarrow \alpha = \frac{\pi}{2} \quad \frac{d\theta}{dt}(t=0) = -A\frac{\gamma}{2}\cos\frac{\pi}{2} - A\omega\sin\frac{\pi}{2} = -A\omega = C$

$\Rightarrow A = -\frac{C}{\omega} \Rightarrow \theta = -\frac{C}{\omega}e^{-\gamma/2\, t}\cos(\omega t + \frac{\pi}{2}) = \frac{C}{\omega}e^{-\frac{\gamma}{2}t}\sin\omega t$

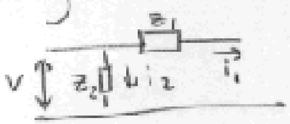


d) $Q = \frac{\omega_0}{\gamma} \Rightarrow e^{-\frac{\gamma}{2}t} = e^{-\omega_0 t/2Q} = e^{-2\pi f_0 t/2Q} = e^{-\pi\frac{t}{T}/Q} =$

$= e^{-\pi n/Q}$ met $n = \frac{t}{T}$ = aantal perioden doorlopen na t seconden $\Rightarrow$ amplitude valt af tot $\frac{1}{e}$ in $\frac{Q}{\pi}$ perioden.

Vraag 2

a)



met $z_1 = i\omega L_{01}$ en $z_2 = \frac{1}{i\omega C_{01}}$

Ga uit van:
$$\begin{cases} v = i_2 z_2 \Rightarrow i_2 = \frac{v}{z_2} \\ dv = -i_1 z_1 dx \rightarrow \frac{dv}{dx} = -i_1 z_1 \\ di_1 = -i_2 dx \rightarrow \frac{di_1}{dx} = -i_2 \end{cases}$$

$$\frac{d^2 i_1}{dx^2} = -\frac{di_2}{dx} = -\frac{1}{z_2}\frac{dv}{dx} = \frac{z_1}{z_2} i_1 \rightarrow \frac{d^2 i_1}{dx^2} = \gamma^2 i_1 \quad \text{met } \gamma = \sqrt{\frac{z_1}{z_2}}$$

b) $$\frac{d^2}{dx^2}(Ae^{-\gamma x} + Be^{\gamma x}) = \frac{d}{dx}(-\gamma Ae^{-\gamma x} + \gamma Be^{\gamma x}) = \gamma^2 Ae^{-\gamma x} + \gamma^2 Be^{\gamma x} = \gamma^2 i$$

$$v(x) = z_2 \cdot i_2(x) = -z_2 \cdot \frac{di_1(x)}{dx} = -z_2 \cdot (-\gamma Ae^{-\gamma x} + \gamma Be^{\gamma x}) = z_{01}(Ae^{-\gamma x} - Be^{\gamma x})$$

met $z_{01} = \gamma \cdot z_2 = \sqrt{\frac{z_1}{z_2}} \cdot z_2 = \sqrt{z_1 z_2} = \sqrt{\frac{L_{01}}{C_{01}}} = \frac{1}{2\pi}\sqrt{\frac{\mu}{\varepsilon}} \ln\frac{r_2}{r_1}$

$$\gamma = \sqrt{\frac{z_1}{z_2}} = i\omega\sqrt{L_{01}C_{01}} = i\omega\sqrt{\varepsilon\mu} \quad \left(= \frac{i\omega}{c_2}\right)$$

c) $v(t) = v(x)_+ e^{i\omega t}$ $\qquad v(x)_+ = z_{01} A e^{-\gamma x} \Rightarrow v(x,t)_+ = z_{01} A e^{i\omega(t - \sqrt{\varepsilon\mu} \cdot x)}$

De snelheid van deze lopende golf is dus $\frac{1}{\sqrt{\varepsilon\mu}}$

d) Randvoorwaarden op $x = 0$: $V_1 = V_2$ (1)

en $I_1 = I_2$ (2)

(1): $z_{01} \cdot (Ae^{-\gamma_1 x} - Be^{\gamma_1 x}) = z_{02}\, Ce^{-\gamma_2 x}$

(2): $Ae^{-\gamma_1 x} + Be^{\gamma_1 x} = Ce^{-\gamma_2 x}$

op $x = 0$, dus e-machten vallen eruit

dus: $\begin{rcases} z_{01}(A - B) = z_{02} C \\ A + B = C \end{rcases}$ $B = \frac{z_{01} - z_{02}}{z_{01} + z_{02}} A$ en $C = \frac{2 z_{01}}{z_{01} + z_{02}} \cdot A$

Voor de spanningsamplitudes van de invallende (i), doorgelaten (d) en weerkaatste (w) spanningsgolven geldt het volgende:

$V_i = z_{01} \cdot A$, $V_d = z_{02} \cdot C$, $V_w = z_{01} \cdot B$.

Dus $V_d = \frac{2 z_{02}}{z_{01} + z_{02}} \cdot V_i$ en $V_w = \frac{z_{01} - z_{02}}{z_{01} + z_{02}} \cdot V_i$

e) $P = \mathrm{Re}(V \cdot I^*)$. voor invallende golf $P_i = z_{01} A^2$

doorgelaten golf $P_d = z_{02} \cdot C^2 = z_{02} \cdot \left(\frac{2 z_{01}}{z_{01} + z_{02}}\right)^2 A^2$

$\Rightarrow \frac{P_d}{P_i} = \frac{4 z_{01} z_{02}}{(z_{01} + z_{02})^2} = \frac{4 \cdot 50 \cdot 75}{125^2} = 0{,}96$, dus 96 % wordt doorgegeven

Opgave 3 G&O '97; tentamen 1 Uitwerking

a) Zie collegesheet Meerlagen-systeem (2)

— b) $kl = \pi/2$, dus $\cos kl = 0$ en $\sin kl = 1$. Dan uit a):

$$\left.\begin{array}{l} 1 + r = -i \frac{n_T}{n_1} t \\ 1 - r = -i \frac{n_1}{n_0} t \end{array}\right\} \begin{array}{l} \text{optellen}: \; t = \frac{2i}{\frac{n_T}{n_1} + \frac{n_1}{n_0}} = \frac{2i\, n_0 n_1}{n_T n_0 + n_1^2} \\ \text{delen}: \; \frac{1+r}{1-r} = \frac{n_T n_0}{n_1^2} \Rightarrow r = \frac{n_T n_0 - n_1^2}{n_T n_0 + n_1^2} \end{array}$$

— c) Invallende & gereflecteerde bundel in n_0, doorgelaten bundel in n_T

$\Rightarrow$ te bewijzen $n_0 |E_0'|^2 + n_T |E_T|^2 = n_0 |E_0|^2$,

oftewel (delen door $|E_0|^2$): $n_0 |r|^2 + n_T |t|^2 = n_0$

Uit b): $n_0 |r|^2 + n_T |t|^2 = \frac{1}{(n_T n_0 + n_1^2)^2} \left[n_0 (n_T n_0 - n_1^2)^2 + 4 n_T n_0^2 n_1^2 \right]$

$= \frac{1}{(n_T n_0 + n_1^2)^2} \left[n_0 (n_T n_0 + n_1^2)^2 \right] = n_0$

— d) Fowles, pag 44: R is de <u>fractie</u> van de invallende lichtenergie die gereflecteerd wordt.

$R = 0$ als $n_T n_0 - n_1^2 = 0 \Rightarrow n_1 = \sqrt{n_T n_0} = \sqrt{1.5} \approx 1.225$

— e) $R = \left[\frac{1.5 - (1.35)^2}{1.5 + (1.35)^2} \right]^2 \approx 0.0094$

— f) Geen coating betekent $n_1 = n_0$ (of ook $n_1 = n_T$)

Dan $R = \left(\frac{0.5}{2.5} \right)^2 = 0.04$ (Volgt ook uit keuze $l = 0$ in -a))

Opgave 4

a $U_p = C \iint e^{ikr} dA$ $\quad r = r_0 + y \sin\theta$

$\Rightarrow U = C e^{ikr_0} \int^{b/2} e^{iky\sin\theta} L\, dy$

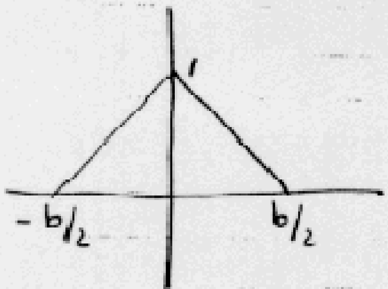
$$= 2 C e^{ikr_0} L \frac{\sin\left(\frac{1}{2} kb \sin\theta\right)}{k\sin\theta}$$

$$r = k\sin\theta \Rightarrow U = C' \frac{\sin \frac{1}{2} br}{r}$$

Normalisatie: $r = 0 \Rightarrow U = 0{,}5\, b \Rightarrow C' = 1$

Dus $U = \frac{1}{2} b \cdot \frac{\sin\left(\frac{1}{2} br\right)}{\left(\frac{1}{2} br\right)}$

b.



$$U(r) = \int_{-b/2}^{b/2} g(y)\, e^{iry}\, dy$$

$$= \int_{-b/2}^{0} \frac{2}{b}\left(y + \frac{b}{2}\right) e^{iry}\, dy + \int_{0}^{b/2} \frac{2}{b}\left(\frac{b}{2} - y\right) e^{iry}\, dy$$

(gebruik gegeven integraal): $= \frac{4}{br^2}\left(1 - \cos\left(r\frac{b}{2}\right)\right)$

c. $I_a = \frac{b^2}{4} \cdot \frac{\sin^2\left(\frac{1}{2} br\right)}{\left(\frac{1}{2} br\right)^2}$

1e secundaire minimum: $\sin\left(\frac{1}{2} br\right) = 0 \ \wedge \ br \neq 0$

$\frac{1}{2} br = \pi \Rightarrow r = 2\pi b$

$$I_b = \frac{16}{b^2 r^4}\left(1 - \cos\left(r\frac{b}{2}\right)\right)^2$$

1e minimum: $\cos\frac{rb}{2} = 1 \ \wedge \ r \neq 0$

$$\frac{rb}{2} = 2\pi$$

$$r = \frac{4\pi}{b}$$

d.

———— : zonderfilter
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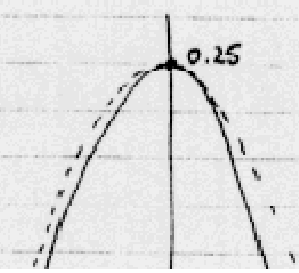

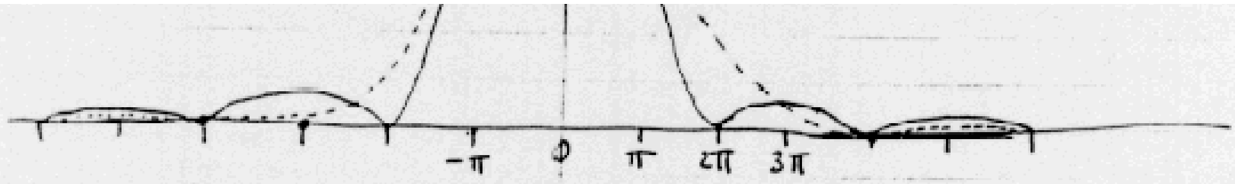

De zijlobben zijn duidelijk minder. Maar het echte eerste maximum zit in de hoofdpiek opgeborgen. Daarom breder.